

# The Costs and Benefits of Caring: Aggregate Burdens of an Aging Population

Finn Kydland<sup>1,2</sup> and Nick Pretnar<sup>2</sup>

<sup>1</sup>*University of California, Santa Barbara*

<sup>2</sup>*Carnegie Mellon University, Tepper School of Business*

June 26, 2018\*

## Abstract

There has been recent attention to the increasing costs to individuals and families associated with caring for people who are afflicted with diseases such as dementia, including Alzheimer's. In this paper we ask, what are the quantitative implications of these trends for important aggregates, including going forward in time. We develop an overlapping generations general equilibrium model that features government social insurance, idiosyncratic old-age health risk, and transfers of time on a market of informal hospice care from young agents to old agents. The model implies that the decline in annual output growth in the United States since the 1950s can be partly attributed to decreases in the working-age share of the adult population. When accounting for the time young people spend caring for sick elders, positive Social Security + Medicare taxes lead to reductions in the growth rate of annual output of approximately 20 basis points. Relative to an economy with no old-age insurance systems, Social Security + Medicare taxes lead to future reductions in output of 6% by 2056 and 17% by 2096. We show that depending on the working-age share of the adult population, eliminating Social Security + Medicare is not necessarily Pareto improving, leaving those afflicted by welfare-reducing diseases worse off. Placed in the context of an aging United States population, these phenomena could have dramatic or muted impacts on future economic outcomes depending on the prevalence rate of high-cost diseases and the rate at which labor is taxed to fund old-age consumption under a pay-as-you-go social insurance system.

---

\*Nick acknowledges support from the National Science Foundation Graduate Research Fellowship under Grant No. DGE1252522. We would also like to thank seminar participants at the University of California - Santa Barbara, Carnegie Mellon University, the University of Missouri, and the Spring 2018, Midwest Macroeconomics Conference at the University of Wisconsin. Special thanks to Laurence Ales, Javier Birchenall, Ali Shourideh, Joe Haslag, and Bill Bednar for their helpful comments.

# 1 Introduction

For the United States and other developed countries, population aging will increase the number of individuals with high cost-of-care diseases such as Alzheimer's and dementia, thus increasing strains on both publicly funded social insurance systems and the financial portfolios of individual families. Caring for infirm elderly requires substantial resources, both in terms of market-traded commodities and off-market time. Recent empirical evidence suggests that many working-age adults spend substantial shares of their available time providing informal care for sick and diseased elders. Few papers have studied this phenomenon, and to our knowledge no paper has examined the general equilibrium impacts of what happens when an aging population demands increasingly higher levels of informal care from its younger relatives. Here, we examine the aggregate economic impacts of these trends in an overlapping generations environment where retired adults face idiosyncratic welfare shocks due to their health status.

We find that social insurance programs in a pay-as-you-go structure crowd out investment, driving young people to spend less time working, which further depletes the stock of capital and drives down long-run growth rates relative to a tax-free environment.<sup>1</sup> In the presence of intergenerational transfers of off-market time from young to old, lifetime welfare increases when social insurance tax rates fall as savings and investment increase. Young agents expect to enjoy being cared for by their offspring when old and plan for this spillover effect when choosing savings. This is because time transfers from young to old on an informal care market can help offset the adverse welfare implications of incomplete markets for insurance against old-age welfare shocks. Yet, while reducing social insurance taxes may increase expected lifetime utility, a reduction is not necessarily Pareto improving if the working-age share of adults is low. This is because old agents afflicted with a welfare-reducing disease are made worse off as taxes fall if labor is elastically supplied and the number of workers is small enough. In various counterfactual simulations we explore the implications of these trends under different population growth rates and different adverse shock probabilities.

Our model helps explain the decline in annual aggregate output growth rates since the 1950s. This should lead researchers to take our findings seriously despite the parsimonious, two-period OLG modeling approach we employ. We argue the results are significant for two main reasons. First, the U.S. population, while relatively younger than those of other OECD countries, is rapidly aging, which could lead to a demographic deficit, a la Japan in the 1990's (Henriksen and Cooley 2018). Second, diseases requiring high levels of hospice care, both on the formal and informal market, are more common amongst older rather than younger individuals. As a motivating example, consider costs associated with dementia, including Alzheimer's, diseases common to older individuals and for which no known cures exist. In 2011 the cost of caring for individuals diagnosed with Alzheimer's and dementia was almost triple that of non-diagnosed individuals (see

---

<sup>1</sup>We affirm findings on the crowding-out effects of Social Security by Hubbard, Skinner, and Zeldes (1995) and Prescott (2004).

Association (2011)). Further, prevalence and cost-of-care for these diseases has increased at faster rates than similar high-cost diseases such as cancer and heart disease (Lepore, Ferrell, and Wiener 2017). In the United States, approximately 70% of these costs are covered by state and federal social insurance programs, while the remaining 30% is covered by family portfolios and younger relatives investing time in caring for older individuals (Hurd et al. 2013). As the population ages larger costs will be born by the individual members of younger generations caring for parents and grandparents.

Of course, other factors contribute to the idiosyncratic risk people face when planning for retirement, not excluding additional health factors. The framework we outline here is general enough to capture all kinds of idiosyncratic risks that affect old-age welfare. Here, shocks act directly on the utility function, with adverse shocks reducing the level of utility from consumption old agents receive. Since we are primarily concerned about health risk, our model fits into the mold of models that incorporate health shocks directly into the utility function, such as DeNardi, French, and Jones (2010), though our specifications are unique to the problems we outline. Thus the types of idiosyncratic risks our model can accommodate do not include those related to wealth and income risks as famously explored in Aiyagari (1994). Our framework can parsimoniously explain how the presence of a caring motive by young agents for old agents can impact aggregate labor supply and other aggregate outcomes. The simple nature of the model — a two-period overlapping generations model — suggests that readers should focus on the qualitative implications of our computational results rather than seek to use them for predictive purposes. The quantitative results we present show the negative impacts of social insurance taxes on long-run growth and the adverse effects an aging population can have on lifetime welfare. These findings can be a useful starting point for considering policy reforms that may dampen the negative long-run impacts of these phenomena.

The paper proceeds as follows. In Section 2 we discuss the population trends and cost estimates associated with the prevalence of high cost-of-care diseases, such as dementia, while also providing an overview of the economics literature on how health shocks affect savings rates and summarizing available data on the allocation of time to care for infirm elders. In Section 3 we outline an OLG model that captures the features discussed in Section 2. In Section 4, we calibrate this model to match observed data points and simulate counterfactuals to understand how population changes affect long-run economic trends. In Section 5 we conclude.

## **2 Background & Discussion**

The primary motivation for our undertaking is to understand how population aging affects aggregate economic outcomes when old members face idiosyncratic risk to old-age welfare. Firstly, there has been a recent emphasis on estimating the costs associated with caring for people who are afflicted with diseases such as Alzheimer’s and dementia (see, for example, Hurd et al. (2013) and Lepore, Ferrell, and Wiener (2017)). Second, while the effects of population aging have been

discussed in many contexts, there have been few studies analyzing general equilibrium outcomes where young people save to insure against idiosyncratic health risk that directly impacts old-age welfare.<sup>2</sup> The closest study that comes to mind is that of Hall and Jones (2007) who model health risk as endogenously affecting survival rates, along with a health status component in utility.<sup>3</sup> Third, to the best of our knowledge nobody has attempted to place idiosyncratic endogenous health risk into a model where young agents provide informal hospice care to ailing loved ones. Our undertaking here thus contextualizes dementia, including Alzheimer's, and other idiosyncratic old-age welfare risk within an economic framework that features a market for informal care.

Alzheimer's disease and dementia impose substantial formal costs on the United States' social insurance system and informal costs on family members tasked with caring for diseased individuals. Total cost estimates for caring for demented elderly individuals range from \$157 to \$215 billion (2010 dollars) depending on the method used to impute time value of informal care (Hurd et al. 2013). Within this range, roughly \$11 billion is covered by Medicare, Medicaid, and Social Security, with the remainder including both out-of-pocket costs paid by afflicted individuals and their families as well as the time value of unpaid, informal care provided by loved-ones (Hurd et al. 2013). For our purposes, "informal care" encompasses all aspects of care which take place off market. "Formal care" will be used to refer to care paid for on the marketplace.<sup>4</sup>

Estimates of total time devoted to informal care for demented persons are not small in magnitude. The Alzheimer's Association estimates that in 2010, 17 billion hours of unpaid care were provided by loved-ones to diseased elders with over 80% of this time burden born by family members. Further, over 90% of those afflicted with Alzheimer's or dementia receive some form of informal care on top of care provided by professional hospice service providers. The spillover effects of shouldering this burden on working-age adults represents an additional societal cost that has not been directly quantified in past studies. Approximately 15% of family members who provide informal care either must take leaves of absence from work or switch from full-time to part-time jobs as a result of the added time burden (Lepore, Ferrell, and Wiener 2017). This fact is not surprising since over half of informal caregivers provide upwards of 21 hours per week of care (Lepore, Ferrell, and Wiener 2017). As the population ages and Alzheimer's and dementia prevalences increase, it is reasonable to expect that the quantity of informal care provided by working-age adults to elderly adults will increase.

There have been no studies, to our knowledge, that estimate the impacts of providing informal care on general equilibrium economic outcomes. This is important because an aging population will likely lead to higher levels of informal care being provided by young people to old people. Several studies have examined how provisions of informal care impact individual labor

---

<sup>2</sup>We are aware of French and Jones (2011), DeNardi, French, and Jones (2010), Edwards (2008), and Palumbo (1999) who look at financial planning decisions within retirement in a partial equilibrium context.

<sup>3</sup>An unpublished study by Azomahou, Diene, and Soete (2009) models health risk as a shock to a health capital stock, as opposed to a direct change in the utility function, which is our approach.

<sup>4</sup>These definitions are consistent with those in Hurd et al. (2013) and Lepore, Ferrell, and Wiener (2017).

force participation and earnings (Muurinen 1986; Carmichael, Charles, and Hulme; Leigh 2010; Van Houtven, Coe, and Skira 2013). Informal caregivers who also participate in the formal labor force supply on average 3 to 10 work hours less per week than their non-caregiving peers (Van Houtven, Coe, and Skira 2013). Providing informal care can thus lead to considerable earnings losses (Muurinen 1986; Van Houtven, Coe, and Skira 2013). Recent applied work suggests that substitution rates between formal nursing home care and informal in-home care depend on states' complex Medicaid reimbursement structures (Mommaerts 2016, 2017). Indeed, paid long-term care and unpaid in-home care are imperfect substitutes (Mommaerts 2017). We conjecture that this imperfection is due to trade-offs faced by younger family members who willingly provide informal care to elders. Since providing off-market care requires a time investment, younger family members must weigh the altruistic benefits they receive from caring for older loved ones versus the loss in lifetime permanent income due to working less. In the data overview below, we show that conditional on providing informal care, individuals work less, sleep less, and have less leisure time. We build these features into the formal, theoretical model in Section 3.

There have been little aggregate data available on the rate at which informal elder care is supplied until recently. In 2011 the American Time Use Survey (ATUS) began asking respondents about time spent engaging in informal care for infirm elders (Bureau of Labor Statistics 2017). These data are available for years 2011 thru 2016, but the number of respondents who participated is small ( $N = 1066$ ). From 2003-2016, ATUS asks respondents how much time they have spent caring for or helping adults, not just the elderly, who require assistance. Weighted averages of time-use for adults age 25-65, where our primary target variable is "adult care", are presented in Table 1.<sup>56</sup>

The impact of increasing Alzheimer's and dementia prevalence on the intensive margin of labor supply thus would seem, at first glance, to be significant in magnitude as the population ages. For illustration, if providing adult care is perfectly substitutable with working then for every 1000 people over the age of 65, 3.55 jobs for individuals under the age of 65 would cease to exist due to the labor supply curve shifting inward. Consider now the aggregate effects of such a change: working less results in a reduction in permanent income, resulting in a reduction in investment, resulting in a reduction in aggregate output and social insurance tax receipts. In the United States the social insurance programs which supplement cost-of-care for infirm elderly are funded as pay-as-you-go (PAYGO) systems, where taxes on labor income paid by the current period's workers fund old-age consumption of the current period's elderly. The aging of the population and the additional costs associated with a population that requires higher levels of hospice care may have substantial impacts on the levels of consumption and thus lifetime welfare future generations may enjoy. Consider, for illustration, in 2010 there were 4.06 persons aged 25-65 for every person age

---

<sup>5</sup>We take the denominator in our weekly time-share calculations to be 112 hours, thus allowing individuals 8 hours of non-allocatable personal time per day.

<sup>6</sup>We choose to use the total "adult care" data point rather than the "elder care" data point due to the small sample size of the latter. Empirical tests show that the differences in weighted averages of both data points are not significantly different from zero. More details are available upon request.

Table 1: Per-Capita Time Allocation of Adults 25 – 65, (ATUS: 2003-2016)

<u>Whole Population, N = 82995</u>			
	Leisure	Labor	Adult Care
Avg. Hrs. per Week	69.642	41.743	0.615
Share of Avg. Total Time*	0.622	0.373	0.005
<u>Provide Positive Off-Market Adult Care, N = 9937</u>			
	Leisure	Labor	Adult Care
Avg. Hrs. per Week	66.150	40.671	5.179
Share of Avg. Total Time*	0.591	0.363	0.046
<u>Provide No Off-Market Adult Care, N = 73058</u>			
	Leisure	Labor	Adult Care
Avg. Hrs. per Week	70.113	41.887	0
Share of Avg. Total Time*	0.626	0.374	0

\* Total time  $7 \cdot (24 - 8)$ .

65 and older (see United Nations: Department of Economic and Social Affairs (2017)). By 2050, this ratio is projected to have fallen to 2.2 and by 2090 to 1.7. Projections by Hurd et al. (2013) suggest that population aging and increasing dementia prevalence by 2040 will cause an increase of 80% from 2010 levels in total societal costs per-capita. This projection is consistent with analyses that show health shocks can weaken the income base, adversely affect savings rates, and lead to a reduction in economic output (Azomahou, Diene, and Soete 2009).

To understand more broadly how long-run declines in aggregate output are related to population aging, in Figure 1 we plot the working-age population ratio ( $wapr$ ), i.e. the ratio of adults age 25-65 to adults age 65 and over, along with the HP-filtered trend of year-on-year GDP growth  $g_Y$  for the United States economy since 1950 (Hodrick and Prescott 1997). When business cycles are removed, the long-run decline in annual GDP growth appears remarkably correlated with the decline in the working-age population ratio. A regression of  $\ln g_{Y,t}$  on  $\ln wapr_t$  reveals that the elasticity of output growth with respect to the working-age population ratio is 1.849, so that a 1% relative increase in workers leads to an approximate 18 basis point increase in the growth rate (see Table 2). Falling  $wapr$  accounts for almost 70% of the decline  $g_Y$  since the 1950s.<sup>7</sup> While this is an extremely parsimonious analysis, the magnitude of this correlation affirms some of the alarm bells sounded recently in Henriksen and Cooley (2018).

Over the same time horizon covered in Figure 1, household savings rates have fallen from

<sup>7</sup>Regression  $R^2 = 0.698$  (see Table 2).

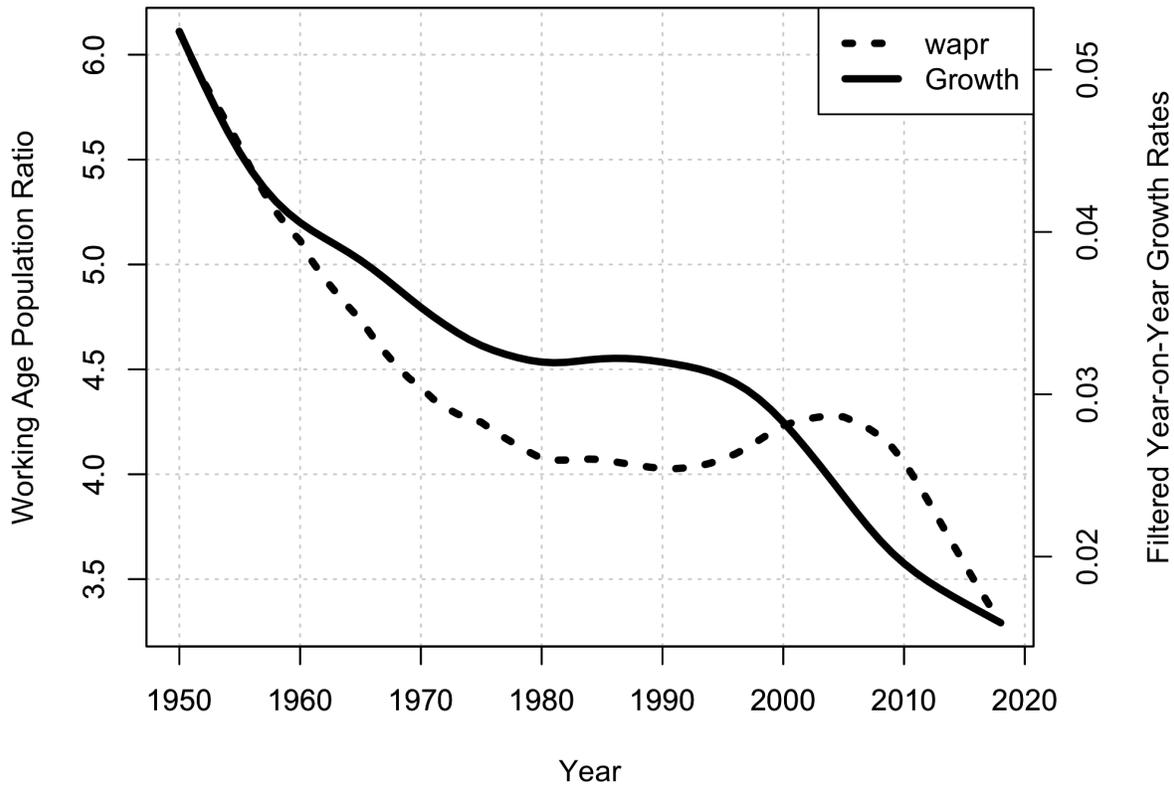


Figure 1: The elasticity of  $g_Y$  with respect to  $wapr$  is 1.849. The decline in  $wapr$  explains about 70% of the decline in year-on-year output growth since the 1950s.

around 10% of disposable income in 1960 to only 5% in 2015. A plethora of literature has examined the distortionary effects and welfare implications of a pay-as-you-go social security system and implications of different tax and benefit structures on the intensive margin of labor and household savings rates (see Prescott (2004), Attanasio and Burgiavini (2003), Huggett and Ventura (1999), and Storesletten, Telmer, and Yaron (1999), for example). A consistent result across these studies is that if labor is supplied elastically, increasing pension taxes on workers does not result in increased pension system revenue but discourages working and can adversely affect the long-run viability of a pay-as-you-go system. In the presence of an aging population where the worker-to-retiree ratio is falling dramatically (again, see Figure 1), one can imagine how such a general equilibrium phenomenon can impact the aggregate economy by reducing labor supply and thus output and individual earnings. Here, we explore these issues when individuals also have a motive to supply informal care to ailing elders.

Our work fits into a broader economic conversation about the role of precautionary savings motives. Particularly, we add to the literature that argues individuals save to insure against illness

in old age (see French and Jones (2011), DeNardi, French, and Jones (2010), and Palumbo (1999)) by endogenizing the decision of young individuals actively to care for their elders, incorporating new dynamics into consumption smoothing. Young households are subsidizing older households both directly thru labor taxes and indirectly by expending time actively to care for them. Thus, more broadly, modeling intergenerational transfers from young to old in this manner, coupled with the perceived guarantee provided by social insurance, could help account for the puzzle of why so many individuals save so little for retirement (see, for example, Benartzi and Thaler (2013) and Hubbard, Skinner, and Zeldes (1995, 1994)). Further, there exists a vein of literature examining European demographic trends that shows that the structure of public pension systems can have an impact on the rates of intergenerational transfers from young to old and vice-versa (Deindl and Brandt 2011; Bonsang 2007; Attanasio and Burgiavini 2003). Designing social insurance systems to better accommodate both future population aging and increases in Alzheimer’s and dementia prevalence could help offset potential welfare loss to future generations due to this phenomenon.

### 3 Model

We consider a model in which agents live a maximum of two periods.<sup>8</sup> Each period, there exists a population of  $N_{yt}$  young households and  $N_{ot}$  old households.<sup>9</sup> There is only one type of young household and two types of old households. Old households can be either diseased  $d_t = 1$  or non-diseased  $d_t = 0$ . All households live a maximum of two periods, though they may die accidentally after their first period of life.<sup>10</sup> For now, assume the population of young households grows at constant rate  $g_N$  so that  $N_{yt}/N_{y,t-1} = (1 + g_N)$ , and the probability that a young household in period  $t$  lives to be an old household in period  $t + 1$  is  $s_{o,t+1}$ .<sup>11 12</sup>

We model the old agents’ consumption process in terms of home production, taking cues from Becker (1965). Young agents can subsidize the home production of diseased old agents’ final consumption by supplying them care time  $h_{ot}$  outside of formal markets. Diseased individuals thus receive flow utility from final consumption  $c_{ot}(d_t = 1)$ , which is produced in the home by using inputs of this off-market care time they receive from their children  $h_{ot}$  and market resources purchased  $x_{ot}(d_t = 1)$ .<sup>13</sup> Meanwhile, their healthy peers only use market resources  $x_{ot}(d_t = 0)$  for production of final consumption because they do not require additional off-market care time from their children. The home production functions we employ for both diseased and non-diseased old

---

<sup>8</sup>We understand that the two-period assumption is rather parsimonious, though we believe the results here to be significant as they stand. Data limitations would mean a less parsimonious model would require more and stronger assumptions. We address this in the concluding section.

<sup>9</sup>Since agents live only two periods, we use  $y$  and  $o$  subscripts to denote their ages.

<sup>10</sup>Young households cannot choose to die, nor can individuals, rather the household is thought to “disappear,” in that all of its members have perished before becoming old.

<sup>11</sup>We will relax the constant growth  $g_N$  assumption in some of our calibration exercises.

<sup>12</sup>The “survival” probability is the probability a young household that enters the economy survives to be an old household next period.

<sup>13</sup>We also refer to  $h_{ot}$  as “hospice” care.

are

$$c_{ot}(x_{ot}(d_t = 1), h_{ot}, d_t = 1) = x_{ot}(1)^{1-\sigma} h_{ot}^\sigma \quad \sigma \in (0, 1) \quad (1)$$

$$c_{ot}(x_{ot}(d_t = 0), d_t = 0) = x_{ot}(0) \quad (2)$$

In Equation (1),  $\sigma$  is the elasticity of final consumption with respect to informal care time supplied. Note that both diseased and non-diseased individuals may purchase hospice care or other health services on the formal market. Such a purchase would fall under market consumption,  $x_{ot}(d_t)$ . Any additional services received by diseased old which are not accounted for on the formal market would fall under informal care-time received,  $h_{ot}$ .

Old households have preferences over consumption that depend on health status  $d_t$ . In this way, the preference structure for old households is similar, though not identical, to that employed in DeNardi, French, and Jones (2010). The form of an old individuals' utility function is chosen to satisfy several conditions. First, we assume that individuals infected with a disease require more resources, both market and off-market, to care for than those who are not. It would be unreasonable to assume that these individuals by consuming more are necessarily better off than their non-diseased peers (after all, they are sick). Let  $u_{ot}(c_{ot}(d_t), d_t)$  denote the flow utility from final consumption for an old individual with disease-status  $d_t$ . This brings us to an assumption about an old individual's utility function.

**Assumption 1.** Suppose  $c_{ot}(1) = c_{ot}(0) = c$ , where  $c$  is any feasible level of final consumption. Then  $u_{ot}(c, 1) < u_{ot}(c, 0)$ . In words, for each level of final consumption, the non-diseased agent always receives higher consumption utility than the diseased agent.

Assumption 1 ensures that it is always better to be non-diseased than diseased. We choose a Stone-Geary flow utility function for diseased old which satisfies this assumption under certain parameter restrictions:

$$u_{ot}(c_{ot}(1), d_t = 1) = \ln(c_{ot}(1) - \nu) \quad (3)$$

$$u_{ot}(c_{ot}(0), d_t = 0) = \ln c_{ot}(0) \quad (4)$$

The flow utility parameterizations in (3) and (4) lead to two basic lemmas.

**Lemma 1.** For all  $\nu > 0$ , Assumption 1 holds.

*Proof.* See Appendix A □

**Lemma 2.** If  $0 < \nu < c_{ot}(1) - c_{ot}(0)$  then the ratio of the marginal utility of non-diseased consumption to diseased consumption is such that  $MU_{ot}(0)/MU_{ot}(1) > 1$ .

*Proof.* See Appendix A □

Lemma 1 is trivial. Lemma 2 says that for certain combinations of the subsistence parameter  $\nu$  and consumption policies, non-diseased agents benefit more from additional final consumption than diseased agents. In our calibration we find that the premise of Lemma 2 holds which is one indicator that in this economy resources are inefficiently allocated in a steady state competitive equilibrium.

Young households use market resources  $x_{yt}$  and leisure time  $l_{yt}$  to produce a final consumption good  $c_{yt}$  according to the production function:

$$c_{yt} = x_{yt}^\gamma \cdot l_{yt}^\mu \quad (5)$$

Young households additionally supply off-market time to care for their elders  $h_{yt}$ , but since this does not affect the final production of their home-produced consumption goods  $c_{yt}$ ,  $h_{yt}$  does not enter into Equation (5). Rather, young households exhibit imperfect altruism toward their elders, discounting the diseased old household's utility at rate  $\eta$ . The flow utility of young households is

$$u_{yt}(c_{yt}, h_{yt}) = \ln c_{yt} + \eta \cdot \ln (c_{ot}(h_{yt}, d_t = 1) - \nu) \quad (6)$$

In addition to consuming and spending time caring for their parents, young agents both supply labor  $1 - l_{yt} - h_{yt}$  and invest  $i_{yt}$  in the market.<sup>14</sup> Young agents earn a before-tax wage rate  $w_t$  and pay social insurance taxes on their income at rate  $\tau_t$ .

Old agents do not work but earn a gross return on their assets  $a_{yt}$  at rate  $R_t$  and also receive Social Security + Medicare transfer benefits from the government  $T_t(d_t)$  which depend on disease status  $d_t$ . For old agents, net outlay must satisfy the budget constraint:

$$x_{ot}(d_t) \leq R_t \cdot a_{yt} + T_t(d_t) \quad (7)$$

At the end of each period, dying young agents leave behind net assets (capital) equivalent to  $a_{y,t+1} \cdot (1 - s_{o,t+1}) \cdot N_{yt}$ . These assets are then distributed evenly and unexpectedly (i.e., "accidentally") as bequests  $b_{y,t+1}$  to young agents entering the economy next period according to

$$b_{y,t+1} = a_{y,t+1} \cdot (1 - s_{o,t+1}) \cdot \frac{N_{yt}}{N_{y,t+1}} \quad (8)$$

Since returns on investment are compounded at the beginning of the period, young agents earn gross return on these assets  $R_t \cdot b_{yt}$ . Having fully-described the right-hand-side of a young agent's budget constraint, their choices of market purchases  $x_{yt}$ , asset-holdings, and labor-supply must satisfy

$$x_{yt} + a_{y,t+1} \leq R_t \cdot b_{yt} + w_t \cdot (1 - \tau_t) \cdot (1 - l_{yt} - h_{yt}) \quad (9)$$

---

<sup>14</sup>Total available time is normalized to 1.

Let  $\psi_t$  denote the share of old population which is diseased. The supply of hospice care by young equals the total amount of hospice care received by diseased old

$$h_{yt} = \frac{N_{ot} \cdot \psi_t \cdot h_{ot}}{N_{yt}} \quad (10)$$

Let  $\rho_t = T_t(1)/T_t(0)$  be the ratio of diseased to non-diseased benefits. The government balances Social Security + Medicare transfers and tax receipts:

$$N_{ot} \cdot (\psi_t \cdot \rho_t \cdot T_t(0) + (1 - \psi_t)T_t(0)) \leq N_{yt} \cdot w_t \cdot \tau_t \cdot (1 - l_{yt} - h_{yt}) \quad (11)$$

Old agents die with certainty at the end of their life and thus absent a bequest motive will choose to consume the entirety of their available cash-on-hand. If  $\delta$  is the depreciation rate of capital then the choice of marginal, period- $t$  investment by young and dis-investment by old must respectively satisfy

$$a_{y,t+1} \leq (1 - \delta) \cdot b_{yt} + i_{yt} \quad (12)$$

$$0 \leq (1 - \delta) \cdot a_{yt} + i_{ot} \quad (13)$$

Young agents do not know whether they will survive to become old and if they do whether they will face a disease that adversely impacts their welfare, but young agents know  $\psi_t$  and how it evolves, just like they know the survival rate. Thus, they make their investment choice both with the aim of smoothing consumption and imperfectly insuring themselves against the adverse welfare-effects of contracting some kind of disease like Alzheimer's or dementia, for example. In this model, given young agents in period  $t$  know the distribution of diseased agents in  $t + 1$ , expectations are perfectly rational. Let  $\beta$  be the time discount factor. In a competitive equilibrium, utility maximizing young agents seek to smooth expected market consumption over the lifecycle according to the expected intertemporal Euler equation:

$$\underbrace{\frac{\gamma}{x_{yt}}}_{MU_y(x)} = \beta \cdot s_{o,t+1} \cdot R_{t+1} \left[ \underbrace{\psi_{t+1} \frac{1 - \sigma}{x_{o,t+1}(1)^{1-\sigma} h_{o,t+1}^\sigma - \nu \left( \frac{h_{o,t+1}}{x_{o,t+1}(1)} \right)^\sigma}}_{MU_o(x(1))} + (1 - \psi_{t+1}) \underbrace{\frac{1}{x_{o,t+1}(0)}}_{MU_o(x(0))} \right] \quad (14)$$

Since the model contains only idiosyncratic uncertainty,  $R_{t+1}$  is pre-determined by the population distribution which is assumed known. Thus young agents choose investment by equating the marginal utility of present market purchases with the discounted expected marginal utility of future consumption given by weighting diseased and non-diseased marginal utilities by the distribution  $\psi_{t+1}$ . The period  $t$  choice of labor supply by young depends on the marginal rate of substitution between consumption and leisure and the marginal rate of substitution between

leisure and off-market care time:

$$\frac{\mu}{l_{yt}} = \frac{\gamma}{x_{yt}} w_t (1 - \tau_t) \quad (15)$$

$$\frac{\mu}{l_{yt}} = \eta \cdot \frac{\sigma}{x_{ot}(1)^{1-\sigma} h_{ot}^\sigma - \nu} \left( \frac{h_{ot}}{x_{ot}(1)} \right)^{\sigma-1} \frac{N_{yt}}{N_{ot} \psi_t} \quad (16)$$

The supply-side of our economy is standard and represented by a single firm which produces an investment good  $I_t$  and a market good  $X_t$ . We assume period  $t$  production is Cobb-Douglas  $F_t(K_t, L_t) = z_t K_t^\alpha L_t^{1-\alpha}$  where  $K_t$  and  $L_t$  are aggregate capital and labor respectively. The rate of return on assets  $R_t$  and before-tax wages  $w_t$  are determined by the marginal products of capital and labor respectively:

$$R_t = 1 + z_t \cdot \alpha \cdot \left( \frac{L_t}{K_t} \right)^{1-\alpha} - \delta \quad (17)$$

$$w_t = z_t \cdot (1 - \alpha) \cdot \left( \frac{K_t}{L_t} \right)^\alpha \quad (18)$$

Finally, total factor productivity  $z_t$  grows at constant net rate  $g_z$ .

### 3.1 Competitive Equilibrium with Transfers

Given  $g_N$ ,  $g_z$ , and exogenously specified sequences of probabilities for disease contraction and survival from young to old,  $\{\psi_t, s_{o,t+1}\}_{t \geq 0}$ , a competitive equilibrium with transfers consists of:

- i. Sequences of policies for consumers:  $\left\{ x_{yt}, l_{yt}, h_{yt}, a_{y,t+1}, x_{ot}(1), x_{ot}(0) \right\}_{t \geq 0}$ .
- ii. Sequences of prices  $\{R_t, w_t\}_{t \geq 0}$ .
- iii. Government policies  $\{T_t(1), T_t(0), \tau_t\}_{t \geq 0}$ .

such that

- a. Young agents' choices satisfy (9) and (14) thru (16).
- b. Old agent consumption policies satisfy (7).
- c. Asset return rates and wage rates are (17) and (18).
- d. Formal and informal markets clear.
- e. The government's budget is balanced.

### 3.2 Steady State and Balanced Growth Path

To solve for a steady state, we assume a balanced growth path (BGP) and de-trend the model accordingly. For now, suppose  $\tau_t = \tau$  is exogenously fixed.<sup>15</sup>

<sup>15</sup>We will revisit this assumption when we discuss potential Pareto improving policies in Section 3.3.

**Proposition 1.** Assume the survival rate  $s_{o,t+1} = s_o$  is constant and the diseased old distribution  $\psi_t = \psi$  is stationary. Then along a BGP the working-age population ratio  $wapr$  is constant and given by

$$wapr = \frac{1 + g_N}{s_o} \quad (19)$$

*Proof.* See Appendix A □

Each period aggregate capital in the economy is only affected by young agents' choice of investment from last period since surviving old agents consume their entire portfolio. Along a BGP aggregate capital and labor are then:

$$K_t = N_{y,t-1} a_{yt} = \frac{N_{yt}}{1 + g_N} a_{yt} \quad (20)$$

$$L_t = N_{yt}(1 - l_{yt} - h_{yt}) \quad (21)$$

Normalize  $N_{ot} = 1$  for period  $t$ , so that  $N_{yt} = wapr$  and use tildes to denote de-trended variables such that de-trended total market good purchases, for example, can be written  $\tilde{X}_t = \frac{X_t}{(1+g_z)^t(1+g_N)^t}$ . Given the normalization  $z_0 = (1 + g_N)^\alpha$ , the de-trended aggregate resource constraint is:

$$\tilde{X}_t + (1 + g_z)(1 + g_N)\tilde{K}_{t+1} \leq (1 - \delta)\tilde{K}_t + \tilde{K}_t^\alpha L_t^{1-\alpha} \quad (22)$$

Let us now consider a steady state where all productivity and population trend growth is removed. The propositions below outline important features of the model's steady state and a necessary condition on balanced output growth in the productivity and population growth re-trended model.

**Proposition 2.** In the de-trended economy, steady state outcomes depend only on the population ratio  $wapr$ , not the population levels.

*Proof.* See Appendix A □

**Proposition 3.** Suppose along a BGP labor hours supplied per worker remains constant and asset holdings per worker grow in proportion to output per worker  $y_t$ . Then the net growth rate of output per worker  $g_y$  is

$$g_y = (1 + g_z)^{\frac{1}{1-\alpha}} - 1 \quad (23)$$

*Proof.* See Appendix A □

**Corollary 1.** Along a BGP, assuming asset holdings per worker grow in proportion to output per worker, the net growth rate of aggregate output is:

$$g_Y = (1 + g_N)(1 + g_y) - 1 \quad (24)$$

Thus as long as  $s_o$  and  $g_N$  are constant, the economy grows at a constant rate.

*Proof.* See Appendix A □

Proposition 3 provides a useful baseline to compare aggregate growth rates under different counterfactual simulations. Clearly, the assumption that  $wapr$  is constant is unrealistic in practice, as we see that  $wapr$  has been falling over time and is projected to continue falling. This fact begs the question as to whether the U.S. economy in the 21st century is in fact on a balanced growth trajectory or rather is exhibiting structural change due to forces like population aging and potentially associated idiosyncratic welfare risk affecting long-run growth rates. In Section 4 we undertake growth accounting exercises under different counterfactual simulations in an attempt to answer this important question and shine a light on potential factors buffering long-run economic growth.

### 3.3 Pareto Improving Social Security + Medicare Taxation

Can the government facilitate Pareto improvements relative to the given 2016 steady state by choosing a tax rate that maximizes social welfare? In Section 4 we show that for big enough  $wapr$  a competitive equilibrium steady state under the observed 2016 Social Security + Medicare tax rate is sub-optimal. The definition of optimality we will employ assumes the government is “dynamically ignorant,” adjusting taxes and transfers by maximizing a static social welfare function  $\mathcal{W}$  assuming the economy is in a 2016 competitive steady state:<sup>16</sup>

$$\mathcal{W}(\tau) = u_y(c_y^*, h_y^*) + \beta \cdot s_o \cdot \left[ \psi \cdot u_o(c_o^*(1), 1) + (1 - \psi) \cdot u_o(c_o^*(0), 0) \right] \quad (25)$$

Note that by maximizing  $\mathcal{W}$  with respect to  $\tau$ , the government takes the market incompleteness due to the idiosyncratic risk as given. Thus the government takes the ratio of diseased to non-diseased benefits  $\rho$  as given and chooses a permanent tax rate  $\tau^*$  that maximizes  $\mathcal{W}$  subject to the economy being in a competitive equilibrium steady state and the government’s budget constraint binding. Being constrained to competitive steady states, the government can only adjust  $\tau^*$  to change the nature of the steady state.

By optimizing only over a static steady state welfare function the “dynamically ignorant” government is acting in a time inconsistent manner and thus choosing a suboptimal plan. As discussed in Kydland and Prescott (1977, 1980), it is inconceivable that in practice a government would be able to implement a fully dynamic optimal tax scheme anyway. This is because economic agents are rational actors with rational expectations and can thus anticipate policy changes, breaking the dynamic optimality (Kydland and Prescott 1977). It is difficult enough for elected officials each beholden to different constituents with different interests to agree upon policy changes, let alone for these same officials to consider how a policy change will forever affect the incentives

---

<sup>16</sup>We denote all steady state variables with a superscript star, as for example  $c_y^*$ .

of future agents. For this reason, a tax scheme implemented by a welfare maximizing “dynamically ignorant” government is probably closer to what would actually happen if Social Security + Medicare reform ever took place.<sup>17</sup>

## 4 Calibrations

For our calibration we set the period length to 40 years and assume young agents enter the economy at age 25 and turn old at age 65. Our calibration strategy assumes the economy is in steady state in 2016, thus taking the 2016 observed population distribution as the initial steady state distribution. We choose parameters to match a set of carefully selected data moments from around 2016. Specifically, we calibrate to leisure, labor, and hospice care average time shares from the 2003-2016 ATUS data, the personal savings rate from the “PSAVERT” time series (BEA, 2016) which measures personal savings as a percentage of disposable income, the ratio of diseased to non-diseased consumption computed from estimates made by Hurd et al. (2013), the 2016 consumption and investment shares of output (BEA, 2016), and the 2015 U.S. labor and capital income shares from Penn World Table 9.0. Table 3 presents the data moment targets and their simulated model counterparts, while Table 4 presents the calibrated parameter values and their sources.

Our calibration requires a couple of assumptions for identification purposes. First, we exogenously set the benefits ratio  $\rho$  using estimates by Hurd et al. (2013) and Mommaerts (2016) to get  $\rho = 1.923$ .<sup>18</sup> Thus, diseased agents receive almost double the benefits from the government as non-diseased agents. To calibrate to a steady state assuming it has been de-trended from a BGP, we only have to pick two of  $g_N$ ,  $s_o$ , and  $wapr$ , due to Equation (19). We set  $g_N$  to accommodate growth in the young population since 1976 and  $wapr$  to equal the observed population ratio for workers to retirees in 2016.

After calibrating the model to a 2016 steady state and extracting parameter values that give the best fit, we want to understand how well our model predicts different aggregate growth rates when the population is evolving in ways inconsistent with the model’s BGP. Figure ?? shows that the working age population ratio since the 1950s has not been constant and has in fact been falling due both to declining fertility rates and rising survival rates. To understand how well our model can predict different  $g_Y$  off the BGP, we compute a transition path from starting steady states with  $wapr$  equivalent to those observed in 1950, 1960, 1970, 1980, and 1990, simulating toward a terminal steady state with  $wapr = 3.475$  as observed in 2016. We assume period 0 of the model is in one of the old  $wapr$ , and then the economy suddenly changes to  $wapr = 3.475$ , allowing 200 simulated periods to facilitate convergence to the new steady state. We then back out implied annual aggregate output growth over a 40-year period, comparing productivity re-trended output from the first period after the sudden change in working age population ratio to that of  $wapr_j$  for  $j \in \{1950, 1960, 1970, 1980, 1990\}$ . Table 5 compares 40-year annual average

---

<sup>17</sup>Taking this commentary a step further though, it may even be a stretch to consider a government that chooses policies via social welfare maximization at all.

<sup>18</sup>The procedure used to set this parameter is described in detail in Appendix B.

model-predicted output growth rates with the actual annual average growth rates over several different time horizons starting at 1950-2016 and increasing in 10 year increments to 1990-2016. Generally speaking the model predicts slightly higher average annual growth at the beginning of the transition path than what actually occurred. This suggests that our counterfactual growth predictions in Section 4.2 may actually be biased upward, so that future growth may stagnate even more than we predict here.

#### 4.1 Steady State Comparative Statics

Using the calibrated parameters from Table 4 we simulate steady state outcomes under different working-age population ratios, and then analyze what happens when the government chooses an optimal tax  $\tau^*(wapr)$  so as to maximize Equation (??) under different values of  $wapr$ .<sup>19</sup> We find that for the 2016  $wapr = 3.475$  the optimal value of  $\tau^*(wapr)$  which maximizes  $\mathcal{W}(\tau, wapr)$  subject to standard competitive equilibrium implementability constraints is zero, so that agents are better off if the social security program is eliminated altogether. As Figure 2 demonstrates, this is not necessarily true if  $wapr$  is small enough, though the magnitude of optimal taxes is far below the 2016 actual rate of 15.3%. For example, optimal steady state tax rates are greater than zero if  $wapr < 2.167$ , peaking at 0.5% when  $wapr = 1.819$  and falling off thereafter. Choosing optimal taxes in this manner improves social welfare for all values of  $wapr$ , though is only Pareto improving for  $wapr > 1.863$  which we demonstrate in Figure 3. This is because for small values of  $wapr$ , diseased agents are made worse off by reducing the tax rate from the current 2016 value of  $\tau = 0.153$ , though under the 2016  $wapr = 3.745$ , reducing taxes to zero leads to steady state Pareto improvements. The fact that aggregate welfare can be relatively improved but this improvement is not Pareto in nature is a direct consequence of the incomplete markets assumption: agents lack the ability to insure against an idiosyncratic disease shock. If the population is more concentrated amongst old agents, lowering  $\tau$  from the 2016 rate to  $\tau^*(wapr)$  adversely affects the market consumption of diseased old agents when there are not enough young agents to make up for this difference by supplying more  $h_y^*$ .

Under  $\tau^*(wapr)$  young agents save more, work more, enjoy higher wages, and supply more hospice time on the informal market. Output is higher, but rates of return on assets are lower due to an increased capital level driven by higher savings rates. Figure 4 compares steady state values of economic aggregates and agent-level choices for select variables under both  $\tau = 0.153$  and  $\tau^*(wapr)$ . Under both scenarios, aggregate labor supply is increasing in  $wapr$ , though substitution effects drive individual households to enjoy more leisure time and supply less labor when  $wapr$  is high. Still, aggregate output goes up in  $wapr$ , driven by both larger capital stocks and aggregate labor supply.  $R^*$  is increasing in  $wapr$  under both tax regimes, encouraging higher savings. The consequences of implementing  $\tau^*(wapr)$  as  $wapr$  increases are that individuals enjoy both more leisure time and more capital income.

---

<sup>19</sup>Appendix B.0.2 presents the steady state equations.

## 4.2 Growth Under Different Counterfactual Regimes

One goal of this project is to understand how the welfare risk of contracting a debilitating old-age disease like Alzheimer’s or dementia may affect future aggregate output growth when the population is aging. As a baseline, we follow techniques described in Krueger and Ludwig (2007) to simulate a transition path between the calibrated 2016 steady state and a far-off future steady state assuming the population converges after 200 periods to the United Nations predicted, 2096 median-variant population distribution (see United Nations: Department of Economic and Social Affairs 2017). We then examine projected growth rates and lifetime welfare under the following policy reforms. First, we consider how the economy evolves when the “dynamically ignorant” government chooses  $\tau^*$  as a function of the present population distribution, and households are surprised by this change, failing to anticipate it.<sup>20</sup> Second, we consider a policy reform where the government decides to begin fully reimbursing working-age adults for their off-market time at the before-tax market wage.<sup>21</sup> Finally, we simulate a dynamic transition path under unexpected shocks to the disease risk rate. We consider growth under a cure by 2056 and a cure by 2096, as well as growth under 10%, 20%, 50%, and 100% increases in cross-sectional risk by 2096.<sup>22</sup> For computational reasons, we assume changes are permanent after 2096 so the economy has some steady state outcome to which to converge.

Table 6 presents simulated average annual aggregate output  $Y_t$  growth rates  $g_Y$  over periods of modulus 40 beginning in 2016. For the baseline simulations holding  $\tau$  and  $\rho$  at their observed and calibrated 2016 values, we compare the dynamic transition path of an economy aging according to U.N. projections with the BGP predicted growth rate from Proposition 3 and Corollary 1. Our regime-change counterfactuals operate as follows. First, we suppose that the economy is in the initial 2016 steady state, then the regime change occurs suddenly. This means that the government unexpectedly changes the tax rate or introduces a reimbursement scheme, or the disease contraction rate changes unexpectedly. When the government is involved, we assume that it chooses  $\tau$  given the economy is in the 2016 steady state and thinking it will stay there, that is we do not assume that anybody has taught the government the fundamentals of dynamic programming. For all of these changes, in period  $t = 2$  right after the 2016 steady state, the economy has changed unexpectedly, but agents have not updated their dynamic plans. We thus simulate the economy for 200 periods to allow for it to converge to the new steady state, which generally happens after only 8-10 periods anyway. All of our counterfactual simulations occur off the BGP, where the population distribution is evolving exogenously according to U.N. estimates.

<sup>20</sup>Here, we consider the 2016  $wapr = 3.475$  as “present.”

<sup>21</sup>Under this reform, the young agent’s budget constraint is:

$$x_{yt} + a_{y,t+1} \leq R_t \cdot b_{yt} + w_t(1 - \tau_t)(1 - l_{yt} - h_{yt}) + w_t \cdot h_{yt} \quad (26)$$

while the government faces budget constraint:

$$N_{ot} \cdot (\psi_t \cdot \rho_t \cdot T_t(0) + (1 - \psi_t)T_t(0)) \leq N_{yt} \cdot (w_t \cdot \tau_t \cdot (1 - l_{yt} - h_{yt}) - w_t \cdot h_{yt}) \quad (27)$$

<sup>22</sup>For a “cure” we consider a situation where  $\psi$  drops to 0.0001 to ensure Inada conditions hold.

From these simulations it is apparent that an aging population substantially reduced growth relative to a BGP. In fact this reduction is on the order of 130 basis points annually, leading to compounded aggregate output losses of 38% by 2056 and 67% by 2096 relative to an economy where  $wapr$  held constant at 2016 levels. The “dynamically ignorant” government can facilitate higher growth by eliminating the Social Security + Medicare system. This does not adversely affect welfare and in fact generates Pareto improvements for the dynamic transition path throughout the entire 21<sup>st</sup> century (see Figure 5). Implementing a before-tax reimbursement scheme has similar, positive effects on growth, though the difference relative to the baseline is marginal. Still, reimbursing young agents at the market rate for their supply of  $h_{yt}$  is Pareto improving, which can also be seen in Figure 5. Finally, achieving a full cure  $\psi = 0.0001$  by 2056 would have a small impact on growth, increasing  $g_Y$  by about 10 basis points, though this impact is not as large as that of a tax elimination. Long-run growth rates are decreasing in risk  $\psi$ , though only marginally. As would be expected, long-run welfare is also decreasing in  $\psi$ , though we do not plot this here.

## 5 Conclusion

Including both idiosyncratic health risk and a motive for young people to engage in informal care of their elders allows the standard, two-period overlapping generations model with production and social insurance taxes to broadly describe the observed decline in aggregate output growth since the 1950s. The model we present qualitatively describes and matches the observed tradeoffs from the ATUS data that agents face when making a decision to provide time on the informal market. These results are important because they should encourage researchers to take seriously the model’s predictions about future economic outcomes in an environment with a rapidly aging population. High social insurance taxes weigh negatively on personal investment and labor supply, driving down output, and adversely affecting welfare of all agents for a large enough working-age population ratio. Due to incomplete markets, the rate at which the population ages can adversely impact lifetime welfare of diseased agents when not enough young agents are alive to supply informal care. In counterfactual simulations, policies like reimbursement of informal care time and elimination of Social Security + Medicare taxes improve both growth and welfare over the U.N.’s medium-variant projected population distribution throughout the 21<sup>st</sup> century. These results should encourage policy makers to consider how the age-distribution and idiosyncratic risk affect economic aggregates when proposing reforms to address stagnating growth and under-funded pension systems.

While it has become common for economic researchers to consider population dynamics within many-period OLG models (see for example Ríos-Rull (2001), Krueger and Ludwig (2007), Backus, Cooley, and Henriksen (2014), and Henriksen and Cooley (2018), among others), a lack of widely available data on welfare-reducing disease risk and age-specific informal-care supply has led us to stick to this simpler version. Incorporating more age-heterogeneity would not be too difficult, but economically (and behaviorally) data limitations would mean we would be forced to make

many more stronger assumptions without adequate empirical justification. We feel our parsimonious approach is adequate to underscore the main point of the analysis: when including old-age idiosyncratic welfare risk and a market for informal care, population aging can discourage working and thus saving, phenomena which are exacerbated by high social insurance taxes and lead to long-run aggregate output and lifetime welfare reductions.

Table 2: OLS Estimate of the Elasticity of  $g_Y$  w.r.t.  $wapr$

<i>Dependent variable:</i>	
Log of Filtered Growth	
$\ln(wapr)$	1.849*** (0.148)
Constant	-6.202*** (0.219)
Observations	69
R <sup>2</sup>	0.698

*Note:* \*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Table 3: Calibration Targets

Moment	Model	Data	Source
$l_y^*$	0.617	0.622	ATUS, 2003-2016 Avg.
$1 - l_y^* - h_y^*$	0.375	0.373	ATUS, 2003-2016 Avg.
$h_y^*$	0.008	0.005	ATUS, 2003-2016 Avg.
Savings Rate	0.137	0.049	BEA, 2016
$x_o^*(1)/x_o^*(0)$	1.315	1.360	Hurd et al. 2013
$X^*/Y^*$	0.812	0.680	Consumption Share, 2016
$I^*/Y^*$	0.188	0.320	Investment Share, 2016
Labor Income Share	0.645	0.600	Penn World Table 9.0, 2015
Capital Income Share	0.355	0.400	Penn World Table 9.0, 2015

Table 4: Calibrated Parameter Values

	Value	Source
$g_z$	0.7439	Non-Farm MFP, 1.4% Annual Growth (1950-1990)
$g_N$	0.660	Growth in Age 25-65 Pop. (1976-2016)
$\delta$	0.9517	40 years of 6% annual depreciation
$\alpha$	0.35	Post-war avg. capital share (DeJong and Dave 2011)
$\beta$	0.4457	Annual discounting of 0.98 over 40 years
$\gamma$	0.373	ATUS (2003-2016) avg. work
$\mu$	0.622	ATUS (2003-2016) avg. leisure
$\eta$	0.005	ATUS (2003-2016) avg. adult care
$wapr$	3.475	U.S. Working-age pop. ratio 2016 (UN)
$\tau$	0.153	S.S. + Medicare tax rate 2016
$\rho$	1.923	Ratio of diseased/non-diseased benefits (see Appendix B.0.1)
$\psi$	0.14	Risk of contracting dementia (see Hurd et al. 2013)
$\nu$	0.099	Subsistence of old: calibrated to match data
$\sigma$	0.659	Intensity of $h_{ot}$ in home production: calibrated to match data

Table 5: Model Performance: Predicted Avg. Annual Growth

Year	Starting $wapr$	Model $g_Y$	Observed $g_Y$	Data Period
1950	6.111	0.035	0.031	1950-2016
1960	5.114	0.032	0.030	1960-2016
1970	4.414	0.030	0.027	1970-2016
1980	4.075	0.027	0.026	1980-2016
1990	4.028	0.025	0.023	1990-2016

Table 6:  $g_Y$  Under Different Regimes

Model	Population	Predicted Avg. Annual Growth $g_Y$		
		2016-2056	2016-2096	2016-2136
Baseline	BGP	0.035	0.035	0.035
Baseline	Transition	0.022	0.021	0.019
Optimal $\tau^*(wapr = 3.475) = 0$	Transition	0.024	0.023	0.021
Reimbursement of $h_{yt}$ at $w_t$	Transition	0.022	0.021	0.019
$\psi_{2056} = 0.0001$	Transition	0.023	0.021	0.019
$\psi_{2056} = 0.07, \psi_{2096} = 0.0001$	Transition	0.022	0.021	0.019
$\psi_{2056} = 0.146, \psi_{2096} = 0.154$	Transition	0.022	0.020	0.019
$\psi_{2056} = 0.154, \psi_{2096} = 0.168$	Transition	0.022	0.020	0.019
$\psi_{2056} = 0.175, \psi_{2096} = 0.210$	Transition	0.022	0.020	0.019
$\psi_{2056} = 0.21, \psi_{2096} = 0.280$	Transition	0.022	0.020	0.018

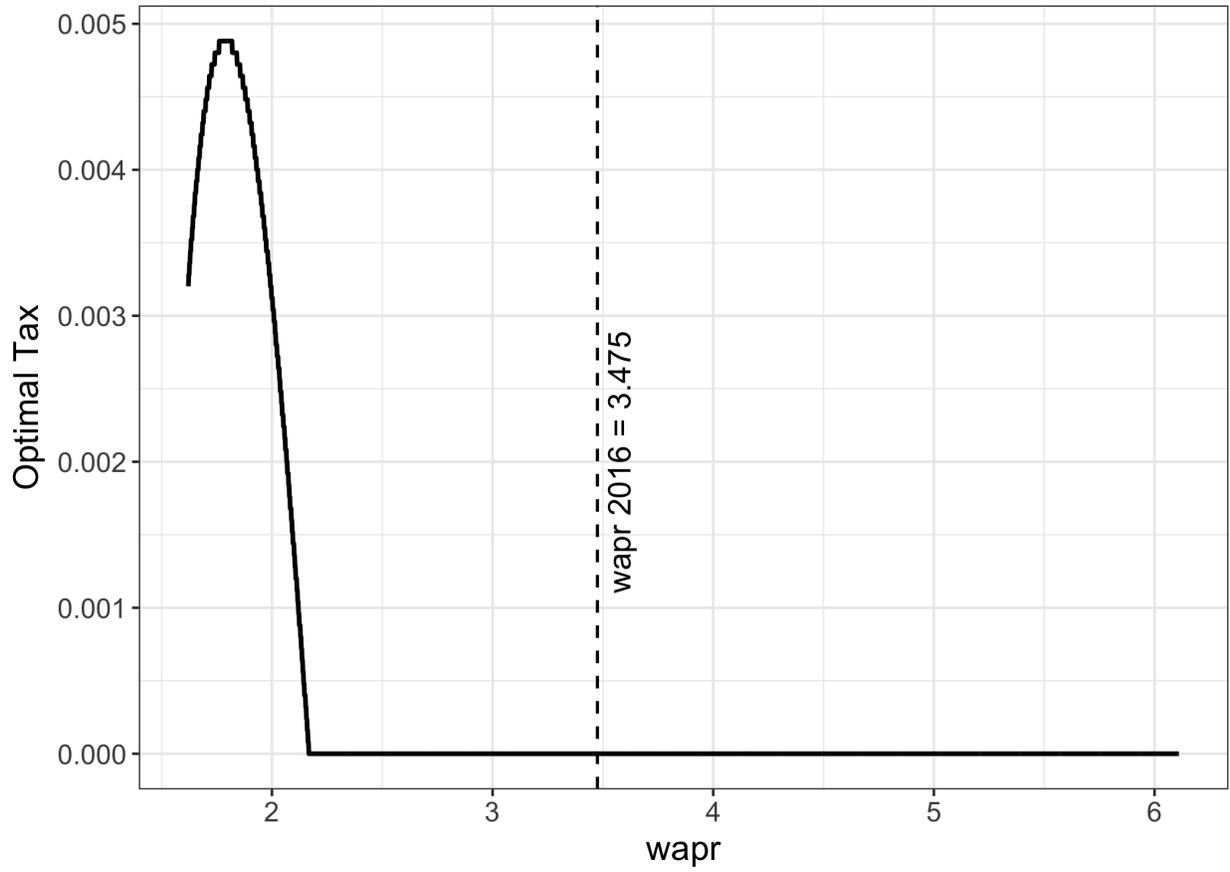


Figure 2: Here we plot  $\tau^*(wapr)$ , the solution to maximization of  $\mathcal{W}(\tau, wapr)$  under different values of  $wapr$ .

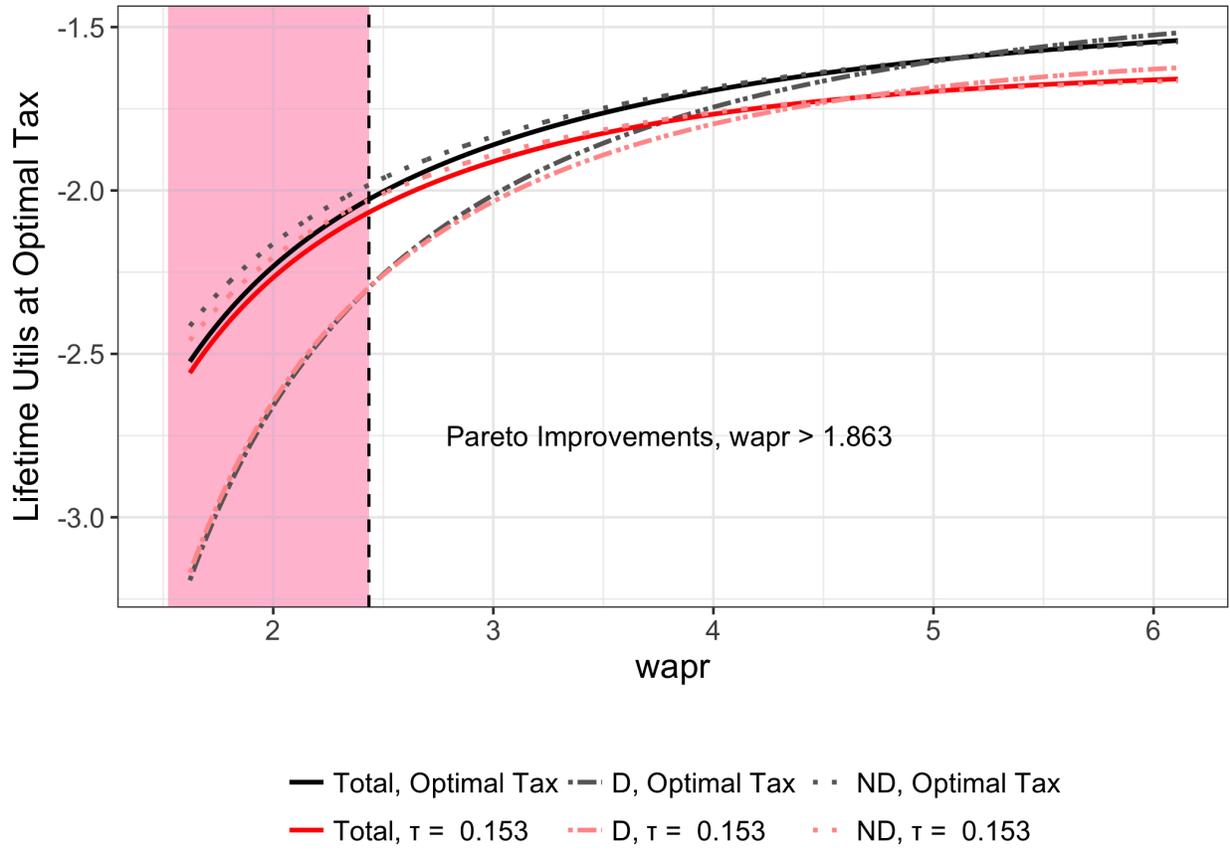


Figure 3: Black lines represent utility under optimal taxes, and red lines utility under  $\tau = 0.153$ , the 2016 Social Security + Medicare tax rate. Maximizing a static social welfare function can lead to steady state Pareto improvements for big enough  $wapr$ , i.e. to the left of the shaded pink region.

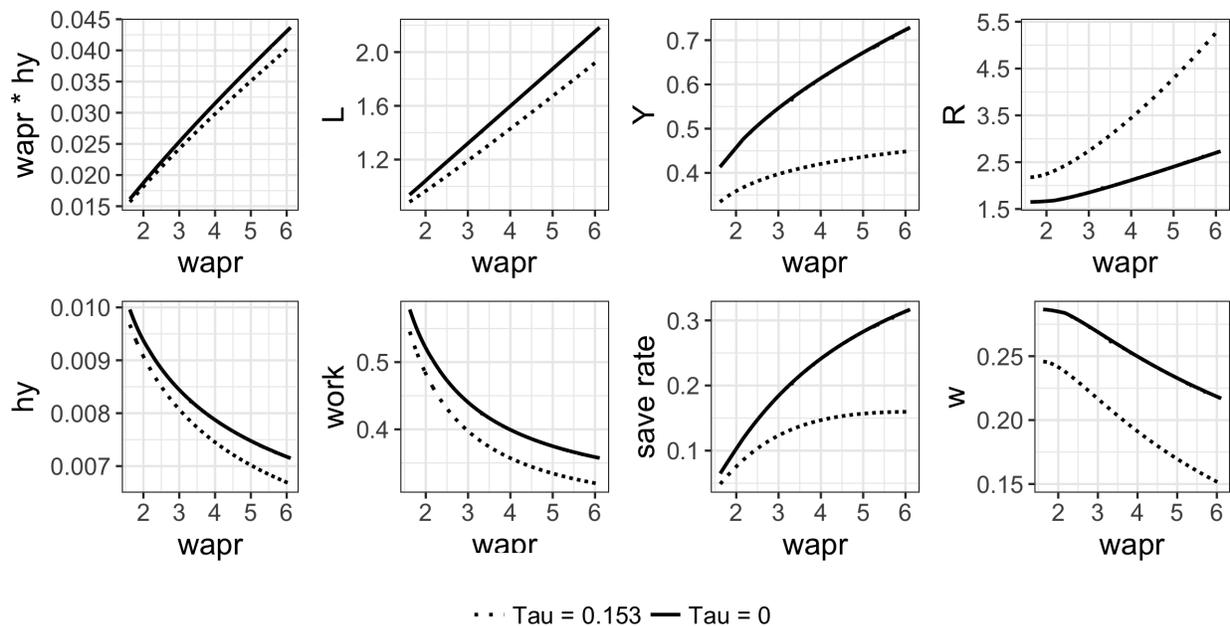


Figure 4: Here we plot steady state outcomes as a function of  $wapr$  and different tax rates. Solid lines represent economic variables when the government chooses  $\tau^*(wapr)$  to maximize  $\mathcal{W}$ , and dotted lines represent variables under  $\tau = 0.153$ , the 2016 Social Security + Medicare tax rate.

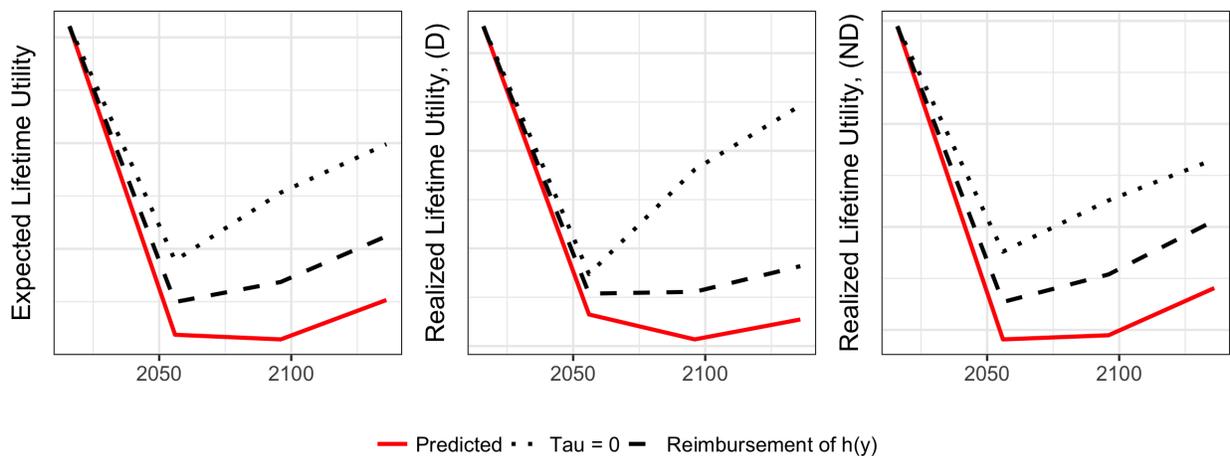


Figure 5: Eliminating the Social Security + Medicare system or implementing a reimbursement scheme holding  $\tau = 0.153$  fixed would create dynamic Pareto improvements over the 21<sup>st</sup> century.

## A Proofs

**Lemma 1.** For all  $\nu > 0$ , Assumption 1 holds.

*Proof.* This proof is trivial, but requires the assumption that  $c > \nu$  so that Inada conditions are satisfied and  $c$  is thus feasible. Under that assumption, clearly  $\ln(c - \nu) < \ln c$ .  $\square$

**Lemma 2.** If  $0 < \nu < c_{ot}(1) - c_{ot}(0)$  then the ratio of the marginal utility of non-diseased consumption to diseased consumption is such that  $MU_{ot}(0)/MU_{ot}(1) > 1$ .

*Proof.* Assume Inada conditions are satisfied such that  $c_{ot}(1) > \nu$  and  $c_{ot}(0) > 0$ . Given Corollary 1,  $\nu > 0$ . Note that:

$$MU_{ot}(1) = \frac{1}{c_{ot}(1) - \nu} \quad (\text{A.1})$$

$$MU_{ot}(0) = \frac{1}{c_{ot}(0)} \quad (\text{A.2})$$

Rearranging the inequality in the premise we get

$$c_{ot}(0) - \nu < c_{ot}(0) < c_{ot}(1) - \nu \quad (\text{A.3})$$

$$\Rightarrow \frac{1}{c_{ot}(0)} > \frac{1}{c_{ot}(1) - \nu} \quad (\text{A.4})$$

$\square$

**Proposition 1.** Assume the survival rate  $s_{o,t+1} = s_o$  is constant and the diseased old distribution  $\psi_t = \psi$  is stationary. Then along a BGP the working-age population ratio  $wapr$  is constant and given by

$$wapr = \frac{1 + g_N}{s_o} \quad (\text{A.5})$$

*Proof.* The population of young agents entering the economy in period  $t$  is

$$N_{yt} = (1 + g_N)N_{y,t-1} \quad (\text{A.6})$$

The population of old agents evolves according to

$$N_{ot} = s_o N_{y,t-1} \quad (\text{A.7})$$

Substituting for  $N_{y,t-1}$  we can write:

$$\frac{N_{yt}}{N_{ot}} = \frac{1 + g_N}{s_o} \quad (\text{A.8})$$

Note that  $\frac{N_{yt}}{N_{ot}}$  is the working-age population ratio  $wapr$ . The right-hand side of the above does not depend on  $t$ . Thus:

$$wapr = \frac{1 + g_N}{s_o} \quad (\text{A.9})$$

□

**Proposition 2.** In the de-trended economy steady state outcomes depend only on the population ratio  $wapr$ , not the population levels.

*Proof.* Given the normalization  $z_0 = (1 + g_N)^\alpha$  consider the factor prices and accidental bequests in the de-trended economy:

$$\tilde{R}_t = 1 + \alpha \cdot \left( \frac{1 - l_{yt} - h_{yt}}{a_{yt}} \right)^{1-\alpha} - \delta \quad (\text{A.10})$$

$$\tilde{w}_t = (1 - \alpha) \cdot \left( \frac{a_{yt}}{1 - l_{yt} - h_{yt}} \right)^\alpha \quad (\text{A.11})$$

$$b_{y,t+1} = a_{y,t+1} \cdot \frac{1 - s_o}{1 + g_N} \quad (\text{A.12})$$

Now note that the population levels only enter into Equation (16) of the young agent's first order conditions, but we can write this:

$$\frac{\mu}{l_{yt}} = \eta \cdot \frac{\sigma}{x_{ot}(1)^{1-\sigma} h_{ot}^\sigma - \nu} \left( \frac{h_{ot}}{x_{ot}(1)} \right)^{\sigma-1} \frac{wapr}{\psi} \quad (\text{A.13})$$

All young agent decision variables thus depend on the constant population parameters of  $s_o$ ,  $g_N$ ,  $\psi$ , and  $wapr$ . Finally, the government budget constraint can be written:

$$(\psi T_t(1) + (1 - \psi) T_t(0)) \leq wapr \cdot w_t \cdot \tau \cdot (1 - l_{yt} - h_{yt}) \quad (\text{A.14})$$

The right hand side of young and old agents' budget constraints only depends on prices and choices which depend only on  $wapr$ , not population levels  $N_{yt}$  and  $N_{ot}$ . We can thus drop time subscripts and solve for steady state values. □

**Proposition 3.** Suppose along a BGP labor hours supplied per worker remains constant and asset holdings per worker grow in proportion to output per worker,  $y_t$ . Then the net growth rate of output per worker  $g_y$  is

$$g_y = (1 + g_z)^{\frac{1}{1-\alpha}} - 1 \quad (\text{A.15})$$

*Proof.* Output per worker is  $y_t = Y_t/N_{yt}$ . Suppose labor supply per worker is constant. The gross

growth rate in output per worker can be written:

$$\frac{y_t}{y_{t-1}} = (1 + g_z) \left( \frac{a_{yt}}{a_{y,t-1}} \right)^\alpha \quad (\text{A.16})$$

If  $\frac{y_t}{y_{t-1}} = \frac{a_{yt}}{a_{y,t-1}} = (1 + g_y)$  then:

$$(1 + g_y) = (1 + g_z)(1 + g_y)^\alpha \quad (\text{A.17})$$

$$\Rightarrow g_y = (1 + g_z)^{\frac{1}{1-\alpha}} - 1 \quad (\text{A.18})$$

□

**Corollary 1.** Along a BGP, assuming asset holdings per worker grow in proportion to output per worker, the net growth rate of aggregate output is:

$$g_Y = (1 + g_N)(1 + g_y) - 1 \quad (\text{A.19})$$

Thus as long as  $s_o$  and  $g_N$  are constant, the economy grows at a constant rate.

*Proof.* From Proposition 2, output per worker grows at gross rate  $(1 + g_y) = (1 + g_z)^{\frac{1}{1-\alpha}}$ . Then

$$\frac{y_t}{y_{t-1}} = (1 + g_y) \quad (\text{A.20})$$

$$\Rightarrow \frac{\frac{Y_t}{N_{yt}}}{\frac{Y_{t-1}}{N_{y,t-1}}} = (1 + g_y) \quad (\text{A.21})$$

$$\Rightarrow \frac{Y_t}{Y_{t-1}} = (1 + g_N)(1 + g_y) \quad (\text{A.22})$$

$$= (1 + g_Y) \quad (\text{A.23})$$

Thus  $g_Y = (1 + g_N)(1 + g_y) - 1$ . □

## B Calibration

### B.0.1 Setting $\rho$ — Ratio of Diseased to Non-Diseased Benefits

We calibrate  $\rho$  by using estimates from Hurd et al. (2013) and Mommaerts (2016). Mommaerts (2016) uses RAND Health and Retirement Study (HRS) data to estimate median permanent income (\$14,157 in 2010 dollars) of sample respondents over age 65 from 1998-2010. We then use the Social Security Administration's rule of thumb permanent income replacement rate (0.4) to compute the implied Social Security benefits for the median retiree:

$$14,157 \cdot 0.4 = 5,662.80 \quad (\text{B.1})$$

Using HRS data Hurd et al. (2013) estimates that average total annual Medicare spending for demented individuals is \$5,226. To compute a baseline for total benefits received by diseased agents we add \$5,226 to Equation (B.1) to get \$10,888.80. The steady state ratio of diseased to non-diseased benefits is:

$$\rho = \frac{10,888.80}{5,662.80} = 1.923 \quad (\text{B.2})$$

## B.0.2 Steady State Equations

The de-trended steady state conditions are below. In order they are as follows (B.3) thru (B.6) comprise the young agent's intertemporal consumption condition, intratemporal consumption/leisure condition, intratemporal leisure/hospice care condition, and budget constraint. (B.7) thru (B.8) define the old agents' market goods conditions. (B.9) and (B.10) describe the equilibrium factor prices. (B.11) describes the accidental bequest condition and (B.12) is the government budget constraint.<sup>23 24</sup>

$$\frac{\gamma}{x_y^*} = \beta \cdot \frac{1 + g_N}{wapr} \cdot R^* \left[ \psi \frac{1 - \sigma}{x_o^*(1)^{1-\sigma} (h_o^*)^\sigma - \nu} \left( \frac{h_o^*}{x_o^*(1)} \right)^\sigma + (1 - \psi) \frac{1}{x_o^*(0)} \right] \quad (\text{B.3})$$

$$\frac{\mu}{l_y^*} = \frac{\gamma}{x_y^*} w^* (1 - \tau) \quad (\text{B.4})$$

$$\frac{\mu}{l_y^*} = \eta \cdot \frac{\sigma}{x_o^*(1)^{1-\sigma} (h_o^*)^\sigma - \nu} \left( \frac{h_o^*}{x_o^*(1)} \right)^{\sigma-1} \frac{wapr}{\psi} \quad (\text{B.5})$$

$$x_y^* + a_y^* \leq R^* \cdot b_y^* + w^* \cdot (1 - \tau) \cdot (1 - l_y^* - h_y^*) \quad (\text{B.6})$$

$$x_o^*(1) \leq R^* \cdot a_y^* + \rho \cdot T^*(0) \quad (\text{B.7})$$

$$x_o^*(0) \leq R^* \cdot a_y^* + T^*(0) \quad (\text{B.8})$$

$$R^* = 1 + (1 + g_N) \alpha \left( \frac{(1 - l_y^* - h_y^*)}{a_y^*} \right)^{1-\alpha} + \delta \quad (\text{B.9})$$

$$w^* = (1 - \alpha) \left( \frac{a_y^*}{1 - l_y^* - h_y^*} \right)^\alpha \quad (\text{B.10})$$

$$b_y^* = a_y^* \frac{1 - \frac{1+g_N}{wapr}}{1 + g_N} \quad (\text{B.11})$$

$$T^*(0) = \frac{w^* \cdot \tau \cdot wapr \cdot (1 - l_y^* - h_y^*)}{\rho \cdot \psi + 1 - \psi} \quad (\text{B.12})$$

The steady state does not admit a neat closed form analytical solution due to the non-linearities in (B.5). We solve the steady state using Powell's hybrid method.

<sup>23</sup>Note that we substitute  $s_o = \frac{1+g_N}{wapr}$  and take  $z_0 = (1 + g_N)^\alpha$ .

<sup>24</sup>The equations presented here are first-order conditions after composing flow utility with the home production functions.

## References

- Aiyagari, Rao. 1994. "Uninsured Idiosyncratic Risk and Aggregate Savings". *The Quarterly Journal of Economics* 109 (3).
- Association, Alzheimer's. 2011. "Alzheimer's Association Report: 2011 Alzheimer's disease facts and figures". *Alzheimer's & Dementia* 7:208–244.
- Attanasio, Orazio, and Agar Burgiavini. 2003. "Social Security and Households' Saving". *The Quarterly Journal of Economics*.
- Azomahou, Théophile, Bity Diene, and Luc Soete. 2009. "The role of consumption and the financing of health investment under epidemic shocks". *United Nations University Working Paper Series: #2009-006*.
- Backus, David, Thomas Cooley, and Espen Henriksen. 2014. "Demography and low-frequency capital flows". *Journal of International Economics* 92.
- Becker, Gary. 1965. "A Theory of the Allocation of Time". *The Economic Journal* 75 (299).
- Benartzi, Shlomo, and Richard Thaler. 2013. "Behavioral Economics and the Retirement Savings Crisis". *Science* 339.
- Bonsang, Eric. 2007. "How do middle-aged children allocate time and money transfers to their older parents in Europe?" *Empirica* 34.
- Bureau of Labor Statistics. 2017. "American Time Use Survey (ATUS)". Visited on 11/20/2017. <https://www.bls.gov/tus/home.htm>.
- Carmichael, F., S. Charles, and C. Hulme. "Who will care? Employment participation and willingness to supply informal care".
- Deindl, Christian, and Martina Brandt. 2011. "Financial support and practical help between older parents and their middle-aged children in Europe". *Aging & Society* 31.
- DeJong, David, and Chetan Dave. 2011. *Structural Macroeconometrics*. Second. Princeton University Press.
- DeNardi, Mariacristina, Eric French, and John Jones. 2010. "Why Do the Elderly Save? The Role of Medical Expenses". *Journal of Political Economy* 118 (1).
- Edwards, Ryan. 2008. "Health Risk and Portfolio Choice". *Journal of Business & Economic Statistics* 26 (4).
- French, Eric, and John Bailey Jones. 2011. "The Effects of Health Insurance and Self-Insurance on Retirement Behavior". *Econometrica* 79 (3).
- Hall, Robert, and Charles Jones. 2007. "The Value of Life and the Rise in Health Spending". *The Quarterly Journal of Economics*.
- Henriksen, Espen, and Thomas Cooley. 2018. "The Demographic Deficit". *Journal of Monetary Economics* 93.

- Hodrick, Robert, and Edward Prescott. 1997. "Postwar U.S. Business Cycles: An Empirical Investigation". *Journal of Money, Credit, and Banking* 29 (1).
- Hubbard, R Glenn, Jonathan Skinner, and Stephen Zeldes. 1994. "Expanding the Life-Cycle Model: Precautionary Saving and Public Policy". *The American Economic Review* 84 (2): 174–179.
- . 1995. "Precautionary Saving and Social Insurance". *Journal of Political Economy* 103 (2): 360–399.
- Huggett, Mark, and Gustavo Ventura. 1999. "On the Distributional Effects of Social Security Reform". *Review of Economic Dynamics* 2.
- Hurd, Michael, et al. 2013. "Monetary Costs of Dementia in the United States". *The New England Journal of Medicine* 368 (14).
- Krueger, Dirk, and Alexander Ludwig. 2007. "On the consequences of demographic change for rates of returns to capital, and the distribution of wealth and welfare". *Journal of Monetary Economics* 54:49–87.
- Kydland, Finn, and Edward Prescott. 1980. "Dynamic Optimal Taxation, Rational Expectations and Optimal Control". *Journal of Economic Dynamics and Control* 2:79–91.
- . 1977. "Rules Rather than Discretion: The Inconsistency of Optimal Plans". *Journal of Political Economy* 85 (3).
- Leigh, Andrew. 2010. "Informal care and labor market participation". *Labour Economics* 17.
- Lepore, Michael, Abby Ferrell, and Joshua Wiener. 2017. "Living Arrangements of People with Alzheimer's Disease and Related Dementias: Implications for Services and Supports". *Research Summit on Dementia Care (white paper)*.
- Mommaerts, Corina. 2017. "Are Coresidence and Nursing Homes Substitutes? Evidence from Medicaid Spend-Down Provisions". *Working Paper*.
- . 2016. "Long-Term Care Insurance and the Family". *Working Paper*.
- Muurinen, Jaana-Marja. 1986. "The Economics of Informal Care: Labor Market Effects in the National Hospice Study". *Medical Care* 24 (11).
- Palumbo, Michael. 1999. "Uncertain Medical Expenses and Precautionary Saving Near the End of the Life Cycle". *The Review of Economic Studies* 66 (2).
- Prescott, Edward. 2004. "Why Do Americans Work So Much More Than Europeans?" *Federal Reserve Bank of Minneapolis Quarterly Review* 28 (1).
- Ríos-Rull, José-Victor. 2001. "Population Changes and Capital Accumulation: The Aging of the Baby Boom". *The B.E. Journal of Macroeconomics* 1 (1).
- Storesletten, Kjetil, Chris Telmer, and Amir Yaron. 1999. "The risk-sharing implications of alternative social security arrangements". *Carnegie-Rochester Conference Series on Public Policy* 50.

United Nations: Department of Economic and Social Affairs. 2017. "World Population Prospects: The 2017 Revision". Visited on 07/31/2017. <https://esa.un.org/unpd/wpp/Download/Standard/Population/>.

Van Houtven, Courtney Harold, Norma Coe, and Meghan Skira. 2013. "The effect of informal care on work and wages". *Journal of Health Economics* 32.